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# Some inertial range correlators in fully developed turbulence

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Measurements of the correlators $K_1(r)=\langle\Delta u_r(\Delta\varepsilon_r)^2\rangle$ and $K_2(r)=\langle\Delta u_r(\Delta q_r)^2\rangle$ for separations $r$ within the inertial range are executed in a large wind tunnel and in the atmospheric surface layer at $R_\lambda\approx(2.0$–$12.7)\times10^3$. Here $u$, $q$, and $\varepsilon$ denote longitudinal velocity, turbulent kinetic energy, and dissipation rate, respectively, and $\Delta\phi_r=\phi(x)-\phi(x+r)$, $\phi=u,q,\varepsilon$. It is found that the correlators behave as $K_1(r)=-c_1\sigma_u\langle\varepsilon\rangle^2$ and $K_2(r)=-c_2\sigma_u^5 r/\Lambda$ where $\sigma_u$ is the rms value of the longitudinal velocity, and $\Lambda$ is the external turbulence scale. The constants $c_1$ and $c_2$ are found to be independent of both $R_\lambda$ and the flow geometry which agrees with the Yakhot theoretical prediction [Phys. Rev. E **50**, R20 (1994)].

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An adequate description of the small-scale structure of fully developed turbulence still remains a challenging problem in fluid dynamics. In addition to numerous existing models (for review and classification see She [1]), new phenomenological models continuously appear in the literature. Recently, Yakhot [2] suggested a model which conceptually differs from the rest. He assumed that the velocity $\vec{u}$, kinetic energy $q=\frac{1}{2}u_iu_i$, and dissipation rate $\varepsilon=\nu(\partial u_i/\partial x_j+\partial u_j/\partial x_i)^2$ are independent dynamical variables. Here $u_i$, $u=1,2,3$, are velocity fluctuation components in the directions $x_i$, and $\nu$ is the kinematic viscosity. Yakhot then applied dimensional arguments, not to the fluctuations of $q$ and $\varepsilon$, but to their fluxes, which is a straightforward generalization of the Kolmogorov [3,4] ideas. He found that at very high Reynolds numbers the correlators

$$K_1(r)=\langle\Delta u_r(\Delta\varepsilon_r)^2\rangle,\quad K_2(r)=\langle\Delta u_r(\Delta q_r)^2\rangle,$$

which describe fluxes of $\varepsilon^2$ and $q^2$, respectively, behave in the inertial range as

$$K_1(r)=-c_1\sigma_u\langle\varepsilon\rangle^2,$$

$$K_2(r)=-c_2\sigma_u^5\frac{r}{\Lambda}, \tag{1}$$

$$\eta\ll r\ll\Lambda.$$

Here $\sigma_u$ is the rms value of longitudinal velocity, $\eta$ and $\Lambda$ denote the Kolmogorov and the external turbulence scales, respectively, $c_1$ and $c_2$ are constants, $\Delta\phi_r=\phi(x_1)-\phi(x_1+r)$, $\phi=u_1,q,\varepsilon$, and angular brackets denote averaging over $x_1$.

In accordance with the Yakhot [2] considerations, $c_1$ and $c_2$ should not depend on Reynolds number. Note that governing parameters in Eq. (1) are chosen to be the large-scale characteristics $\sigma_u$ and $\langle\varepsilon\rangle$. Such a choice agrees with the general concept of cascade models which consider the dissipation scale not to be a governing parameter in the inertial range. Some results on $K_1(r)$ from a numerical experiment were presented by Borue and Orszag [5]. Our paper reports an experimental study of correlators $K_1(r)$ and $K_2(r)$ in the atmospheric surface layer and in a large wind tunnel at very high Reynolds numbers.

Measurements in the atmospheric surface layer were taken over a nearly homogeneous field near Carpenter, Wyoming (Oncley [6]). The instantaneous horizontal component of the wind velocity was measured from a tower at 7 m above ground level. After preliminary processing, six time series have been chosen from the total record (Praskovsky and Oncley [7]). Four of these series are analyzed in the present work. The second experiment was executed in the large wind tunnel of the Central Aerohydrodynamic Institute (Moscow, Russia). Longitudinal and lateral velocity components were recorded in the mixing layer and in the return channel of the wind tunnel (Praskovsky *et al.* [8]).

The main flow characteristics of the measurements are listed in Table I. The abbreviations ML, RC, and ASL denote the mixing layer, return channel, and atmospheric surface layer, respectively, and numerals after ASL correspond to the sequence of the time series (in accordance with that in [7]). When their meaning is clear, the notations $u\equiv u_1$ and $x\equiv x_1$ are used throughout the paper. $U$ is the mean longitudinal velocity, $\sigma_\phi$ denotes the rms value of any quantity $\phi$. The longitudinal integral scale $L_1$, the Taylor $\lambda$ and Kolmogorov $\eta$ microscales, and the Reynolds number $R_\lambda$, are defined with standard formulas: $L_1=\sigma_u^{-2}\int_0^\infty\langle u(x)u(x+r)\rangle dx$, $\lambda=\sigma_u/\sigma_{\partial u/\partial x}$, $\eta=(\nu^3/\langle\varepsilon\rangle)^{1/4}$, and $R_\lambda=\sigma_u\lambda/\nu$ where it is assumed that $\langle\varepsilon\rangle=15\nu\langle(\partial u/\partial x)^2\rangle$. Other quantities in Table I will be defined later. Taylor's hypothesis was used to convert from temporal to spatial coordinates.

Two surrogates for the instantaneous values of the dissipation rate $\varepsilon$ were used, namely $\tilde\varepsilon_1=15\nu(\partial u_1/\partial x_1)^2$ and $\tilde\varepsilon_2=7.5\nu[(\partial u_1/\partial x_1)^2+0.5(\partial u_2/\partial x_1)^2]$. As was mentioned above, only $u_1(x_1)$ is available in the atmospheric measurements while both $u_1(x_1)$ and $u_2(x_1)$ were simultaneously recorded in the wind tunnel. Thus $K_1(r)$ was estimated by using $\tilde\varepsilon_1$ for the atmospheric data, and by using both $\tilde\varepsilon_1$ and $\tilde\varepsilon_2$ for the wind tunnel data. Results are presented in Fig. 1 in a nondimensional form

$$F_1(r)=-\frac{K_1(r)}{\sigma_u\langle\varepsilon\rangle^2}.$$

One can see that the scatter of the experimental results is

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TABLE I. Main characteristics of analyzed time series.

| Time series | ML | RC | ASL2 | ASL3 | ASL5 | ASL6 |
|---|---|---|---|---|---|---|
| $U$ (m/s) | 7.87 | 10.8 | 6.58 | 8.10 | 12.9 | 14.5 |
| $\sigma_u$ (m/s) | 1.67 | 1.03 | 0.693 | 1.10 | 1.82 | 2.08 |
| $L_1$ (m) | 1.3 | 4.8 | 42 | 51 | 99 | 77 |
| $\langle\varepsilon\rangle$ ($m^2/s^3$) | 1.90 | 0.115 | 0.0235 | 0.0322 | 0.140 | 0.128 |
| $\lambda$ (cm) | 1.8 | 4.6 | 6.5 | 9.0 | 7.0 | 8.3 |
| $10^{-3}R_\lambda$ | 2.0 | 3.2 | 3.3 | 6.9 | 9.2 | 12.7 |
| $\eta$ (mm) | 0.21 | 0.41 | 0.58 | 0.55 | 0.37 | 0.37 |
| $L_1 R_\lambda^{-3/2}/\eta$ | 0.069 | 0.065 | 0.38 | 0.16 | 0.30 | 0.15 |
| IR1 ($r/\eta$) | 20–4000 | 20–7000 | 20–2000 | 20–2000 | 20–4000 | 20–6000 |
| $c_1 \pm \sigma_1$ | 2.62±0.48 | 2.72±0.39 | 2.90±0.42 | 2.42±0.28 | 1.98±0.24 | 2.28±0.43 |
| IR2 ($r/\eta$) | 20–200 | 20–4000 | 20–300 | 20–1000 | 20–2000 | 20–20000 |
| $c_2 \pm \sigma_2$ | 8.37±0.73 | 8.65±1.39 | 8.07±0.48 | 8.23±0.64 | 7.33±0.85 | 6.49±0.63 |

rather large. This is due to poor statistical convergence of the sign-changing correlator $K_1(r)$ which really represents small differences of large values. It is well known (see, for example, Champagne *et al.* [9]) that measurement of a fifth order moment with, say, 5% accuracy requires a sampling time of about $10^7$ integral time scales. For the atmospheric measurements this corresponds to a sampling time about $10^4$ h, which is completely unrealistic. The present time series have a sampling time about 1 h [7]. For this reason the large scatter is not surprising.

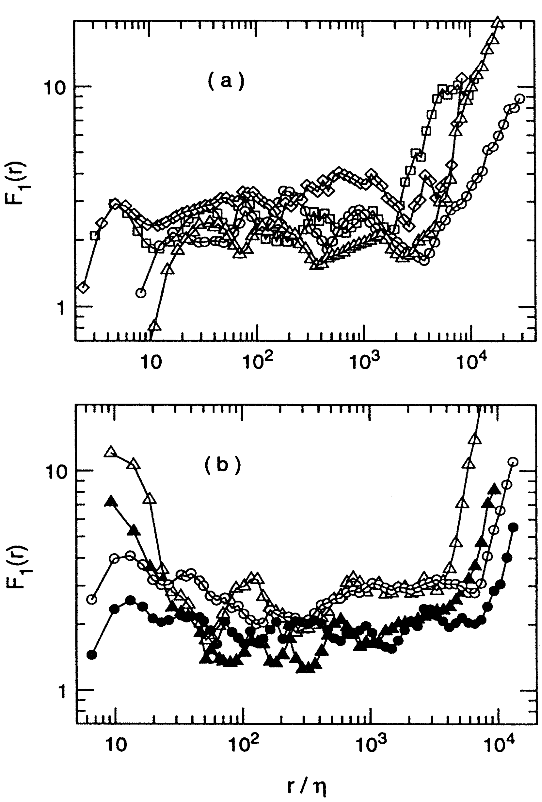


FIG. 1. The nondimensional correlator $F_1(r)$. (a) The atmospheric data, $\varepsilon=\tilde{\varepsilon}_1$: ◇, ASL2; □, ASL3; △, ASL5; ○, ASL6. (b) The wind tunnel data: △, ML; ○, RC. Open symbols, $\varepsilon=\tilde{\varepsilon}_1$; solid symbols, $\varepsilon=\tilde{\varepsilon}_2$.

It is seen in Fig. 1(b) that $K_1(r)$ has the same functional behavior for both $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. This result is expected because $K_1(r)$ is mainly defined by the large values of $\varepsilon$. As was pointed out by Gibson and Masiello [10] (for more results see also Praskovsky [11]), the large values of $\varepsilon$ are well represented by the surrogate $\tilde{\varepsilon}_1$.

It follows from Eq. (1) that $F_1(r)$ should be constant for separations $r$ in the inertial range. In spite of considerable scatter, one can see in Fig. 1 that each curve has a clearly pronounced range of $r$ where $F_1(r)$ is approximately constant. These ranges for $\varepsilon=\tilde{\varepsilon}_1$ are indicated in Table I as IR1. Within these ranges, deviations of $F_1(r)$ from a constant value can be attributed to measurement uncertainty. The mean values $c_1=\bar{F}_1$ and the standard deviations $\sigma_1$ were estimated by averaging $F_1(r)$ over these ranges, and the results are listed in Table I.

The measured values of the correlator $K_2(r)$ are presented in Fig. 2 as

$$F_2(r)=-\frac{K_2(r)}{\sigma_u^5}\frac{\Lambda}{r}.$$

The scale $\Lambda$ was estimated as $0.15\eta R_\lambda^{3/2}$, as will be explained below. The turbulent kinetic energy was estimated by using the one-dimensional surrogate $\tilde{q}_1=\frac{3}{2}u_1^2$. For the wind tunnel data the two-dimensional surrogate $\tilde{q}_2=\frac{3}{4}(u_1^2+u_2^2)$ was also used. Similar to $F_1(r)$, measured values of $F_2(r)$ reveal a considerable scatter. One can also see that results for $\tilde{q}_1$ and $\tilde{q}_2$ are qualitatively similar. In accordance with Eq. (1), the values of $F_2(r)$ should be constant in the inertial range. For each curve in Fig. 2 one can find an extended range of $r$ where $F_2(r)$ is approximately constant, and these ranges for $q=\tilde{q}_1$ are indicated in Table I as IR2. The mean values $c_2=\bar{F}_2$ and the standard deviations $\sigma_2$ obtained by averaging $F_2(r)$ over these ranges are listed in Table I.

Consider the choice of the external turbulence scale $\Lambda$. It is commonly assumed that $\Lambda$ is equal to the turbulence integral scale, which is well represented by the one-dimensional longitudinal scale $L_1$. This assumption works quite well in free flows where there are no large deviations from isotropy even at large scales. However, this is not the case near the flow boundaries, e.g., in the near-wall region of a boundary layer where eddies are strongly expanded in the direction of

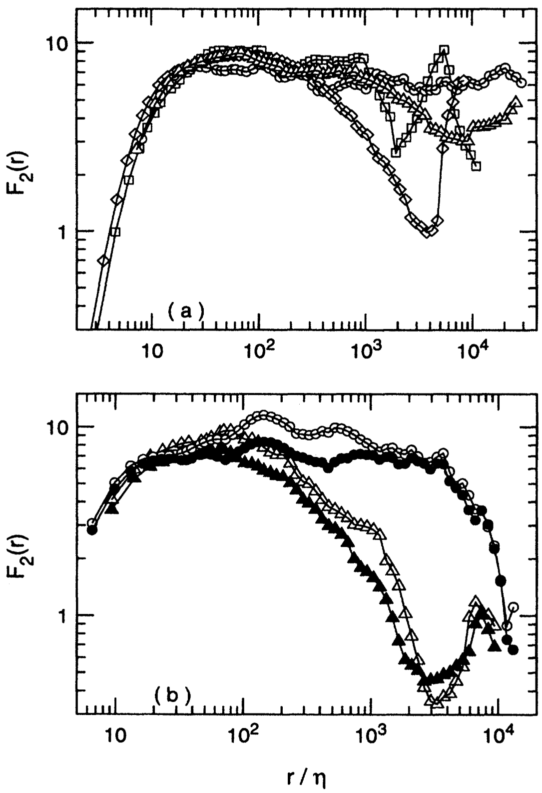


FIG. 2. The nondimensional correlator $F_2(r)$. (a) The atmospheric data, $q=\tilde{q}_1$: ◇, ASL2; □, ASL3; △, ASL5; ○, ASL6. (b) The wind tunnel data: △, ML; ○, RC. Open symbols, $q=\tilde{q}_1$; solid symbols, $q=\tilde{q}_2$.

a mean flow. In particular, in the present atmospheric measurements $L_1$ varies from about 40 to 100 m (see Table I). On the other hand, the integral scale in the vertical direction cannot be larger than the height of the measurements which was 7 m. It follows from these considerations that $L_1$ may not be an adequate measure of the external turbulence scale $\Lambda$. When the present measurements of $K_2(r)$ were nondimensionalized by $L_1$ instead of $\Lambda$, i.e., the value $F_2^*(r)=(L_1/\Lambda)F_2(r)$ was estimated, then $\bar{F}_2^*$ were found to be strongly (and nonsystematically) different for different time series. This variation was assumed to be due to variability of the ratio $L_1 R_\lambda^{-3/2}/\eta$. One can see in Table I that this

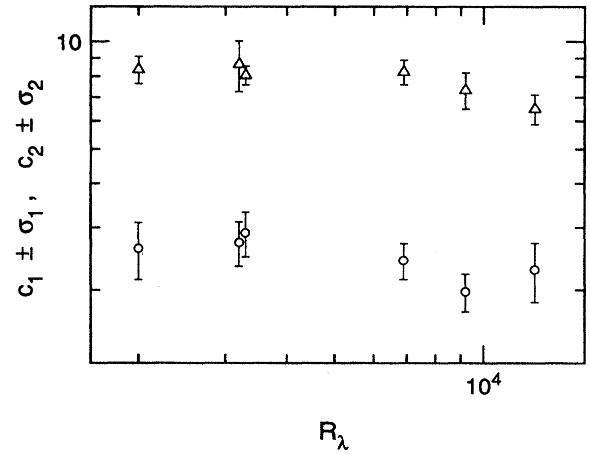


FIG. 3. Dependence of $c_1$ and $c_2$ on the Reynolds number. ○, $c_1$; △, $c_2$.

ratio is about 0.07 in the wind tunnel free flows while it randomly varies from 0.15 to 0.4 in the atmospheric surface layer. To order the present measurements, an estimate $\Lambda = a\,\eta R_\lambda^{3/2}$ was used, and factor $a$ was chosen to be 0.15, which is somewhat arbitrary.

The measured values of constants $c_1$ and $c_2$ versus Reynolds number are presented in Fig. 3. No detectable dependence on the flow geometry is seen, i.e., $c_1$ and $c_2$ for the three analyzed flows (ML, RC, and ASL) seem to depend on $R_\lambda$ in the same way. It is also seen that both $c_1$ and $c_2$ weakly decrease as $R_\lambda$ increases. Taking into account a relatively poor measurement accuracy, we do not think that the present results can provide any definite relation for $c_1(R_\lambda)$ or $c_2(R_\lambda)$. At this stage of study it seems reasonable to assume that both $c_1 \approx 2.5$ and $c_2 \approx 7.5$ are constants for all analyzed time series. Note that $c_2$ depends linearly on the external scale $\Lambda$, which in the present work was chosen to be $0.15\,\eta R_\lambda^{3/2}$.

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